Cilt/Volume:1, Sayı/Issue:1, Yıl/Year:2020, Sayfa/Page: 23-37

Gönderilme Tarihi (Received): 24/01/2020, Düzenlenme Tarihi (Revised):11/03/2020, Kabul Tarihi (Accepted): 23/03/2020, Araştırma Makalesi (Research Article)

STOCK SIMULATION WITH STOCHASTIC MODELS: AN APPLICATION ON ISTANBUL STOCK EXCHANGE

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Abstract

Financial markets are the markets where price changes are experienced depending on the instant changes. That's why, recent studies focus on constant time stochastic models to model stock prices. Geometric Brownian Motion (GBM) is a popular and the most appropriate method to model future prices of stocks, commodity and so on randomly. In this study, usability of GBM model on firms which have operations in main metal industry and Istanbul Stock Exchange. The findings of the study have revealed that GBM is an appropriate model for estimation of stock prices.

Key Words: Geometric Brownian Motion, Stochastic Model, Simulation, Istanbul Stock Exchange, Main Metal Industry

STOKASTİK MODELLERLE STOK SİMÜLASYONU: BORSA İSTANBUL'DA BİR UYGULAMA

Özet

Finansal piyasalar anlık değişimlerin değişimlere bağlı olarak fiyat değişimlerinin yaşandığı piyasalardır. Bu nedenle son yıllarda yapılan çalışmalar, hisse senedi fiyatlarının modellenmesinde sürekli zamanlı stokastik modeller üzerine yoğunlaşmaktadır. Geometrik Brownian Hareket (Geometric Brownian Motion-GBM); hisse senedi, emtia ve benzeri finansal varlıkların fiyatlarının gelecekte alması olası değerlerinin rastsal bir biçimde modellenmesi için kullanılan en uygun ve popüler yöntem olarak kabul edilir. Bu çalışmada, GBM modelinin Borsa İstanbul (BIST)' da işlem gören metal ana sanayi sektöründeki şirketler için kullanılabilirliği değerlendirilmiştir. Çalışma sonucunda elde edilen bulgular, GBM modelinin hisse senedi fiyat tahmini için uygun bir model olduğunu ortaya koymuştur.

Anahtar Kelimeler: Geometrik Brownian Hareketi, Stokastik Model, Simülasyon, BIST, Metal Ana Sanayi.

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Introduction

Nowadays, effect of globalization and improvements on information technologies make economies dependent on each other. Mobile money presents some opportunities and risks for investors, financial institutions, and firms.

No doubt, financial markets and stock markets which present opportunities are important resources of risk and important examples for an uncertain environment. If an investor bought stock with his/her savings, having knowledge about future price of stock or if someone else who wants to utilize his/her money, having knowledge about reasonable choice are important questions for these people. Bachelier who was looking for the answer of the question thought that Brownian motion of Robert Brown is an appropriate model for stock exchange prices in 1900. Brown examined the pollen particles in the suspension under the microscope, observed that they move in a constant irregular manner and called it "Random Motion. Norbert Wiener was the one who laid the mathematical foundations of the model. Essence of Bachelier's dissertation is that assumption of price of stocks is formed randomly. Paul Samuelson who had inspried from Bachelier's dissertation suggested that GBM was more appropriate model for motions of stocks and financial products in 1965. Later, many studies inspired by the suggestion of Paul Samuelson and used it as base. Study of Black and Scholes which is accepted as one of the most significant model for Europian type pricing method, was considered worthy of Nobel price in the field of economy. The Black-Scholes model is not only the basis for further studies on option pricing, but also forms the basis for valuation in many areas such as company valuation, investment decisions, and the value of stock price.

GBM is used as a model that can give us, investors, annual, monthly, daily, hourly and even minute price distributions in the direction of how prices can follow in today's uncertainty environment. In this respect, the aim of our study is to seek answers to the question of what the price of the stock held by a risk-taking investor will be in a given period of time. In order to answer this question, GBM, which is the most commonly used model to randomly model stock price, is used today (Özkan ve Güngör, 2017:393-394).

In this study, non-sample forecast will be made for one month period using GBM model by using daily closing prices of 14 company shares in the metal main industry sector, one of the sectors with the highest stock performance between the dates of 04.01.2010- 30.11.2018.

1. Geometric Brownian Motion

In the financial market, investors compare financial assets with returns rather than prices. The most commonly used basic assumption about the behavior of these returns is that they have a normal distribution. The logarithmic return of the stock at any time t is considered to have a normal distribution. The fact that the stock price will not be negative requires the use of GBM to model the price movement. Furthermore, a lognormally distributed continuous time stochastic process with independent changes can only be explained by GBM. (Demir, 2015:80).

GBM is one of the most commonly used stochastic processes in the financial and economic literature to model the dynamics of stocks and exchange rates. In the financial literature, important models such as the Black-Scholes Option Pricing Model and the Warrant Value Calculation model introduced by Paul A. Samuelson are behind the GBM process (Topper, 2005). GBM may be called “Exponential Brownian Motion” or “Lognormal Diffusion in some studies (Lyuu, 2004: 186). It is more appropriate to model securities prices using the GBM process, a version of the Brownian movement, rather than the Wiener process. Since GBM requires lognormal distribution of returns, returns are prevented to be negative (Önalan, 2007: 204)

Change of stock prices in constant time can be modelled like that;

$$dS = S\left[\mu dt + \sigma dz\right] \quad (1)$$

μ and σ are the annual expected rate of return of stock and the volatility of return. As can be seen from this model, the change in stock price consist of;

- In an infinitely small time period, the expected annual return of the stock and
- In an infinitely small time period, the expected volatility of stock returns consists of the sum of its components (Önalan, 2007:205).

Alike the irregular movement of the Brownian particle in liquid, the prices of financial goods fluctuate randomly with the effect of many economic, political and social factors. As is known, commodity prices are not negative at all and show an exponential growth trend in the long run. This characteristic of price movements can be appropriately modeled by stochastic processes called GBM.

$W = \left(W(t)\right)_{t\geq 0}$ defined as standard Brownian motion, $X = \left(X(t):\ t\geq\ 0\right)$, $X(t) = \mu t + \sigma W(t)$ the process which is completed, is called Trendy Brownian Motion with μ trend (instensity) and σ2 variance parameters. $S = \left(\ S(t);\ t\geq 0\ \right)$ The process can be defined as fallows.

$$S(t) = S_0\ exp\left[X(t)\right] = S_0\ exp\left[\mu t + \sigma W(t)\right] \quad (2)$$

S is called GBM. If $W(0) = 0$, $S(0) = S_0$ should be too. If shows the price of stock for time *t*, for $t_0 < t_1 < \ldots < t_n$.

$$\frac{S(t_i)}{S(t_{i-1})} = exp\left[\mu\left(t_i - t_{i-1}\right)\right] exp\left\{\sigma\left[W\left(t_i\right) - W\left(t_{i-1}\right)\right]\right\} \tag{3}$$

Can be written. In this situation,

$$\frac{S(t_1)}{S(t_0)}, \frac{S(t_2)}{S(t_1)}, \ldots\ldots, \frac{S(t_n)}{S(t_{n-1})} \tag{4}$$

They are independent random variables that show the rate of change of the stock price over time intervals that do not intersect each other. The CPD defining the random behavior of the securities price level *S(t)* is expressed as follows.

$$dS(t) = \mu S(t)\,dt + \sigma S(t)\,dW(t) \tag{5}$$

W shows the standard Brownian movement. *d* is the continuous time in the equation. Using stochastic analysis, the above equation can be rewritten as follows.

$$d\ log\ S(t) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW(t) \tag{6}$$

The logarithmic process is called Arithmetic Brownian Motion. Logarithmic increases in commodity prices are normally distributed. *t* and *u* take the form in the following figure to indicate any two time moments,

$$logS(u) - logS(t) = (\mu - \frac{\sigma^2}{2})(u-t) + \sigma\left(W(u) - W(t)\right)$$
$$\sim N((\mu - \frac{\sigma^2}{2})(u-t), \sigma^2(u-t)) \tag{7}$$

is taken and the following solution is obtained for GBM (Önalan, 2010:253-255).

$$S(T) = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma W(T)\right] \tag{8}$$

If $\{X(t), t \geq 0\}$, $dX = \mu dt + \sigma dZ$ is a Wiener process;

$$\{S(t), t \geq 0\}\ S(t) = e^{x(t)} \tag{9}$$

Is stated as GBM.

The equation of $F(X) = e^{x(t)}$ is solved on the basis of Ito's formula as follows:

$$df = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial X}dX + \frac{\sigma^2}{2}\frac{\partial^2}{\partial X^2}dX^2 \tag{10}$$

Accordingly, when the necessary solutions are made to show the drift parameter in the Brownian motion process and the variance parameter σ2, the GBM defined in continuous time appears as follows (London, 2005:789);

$$dS = (1 + \mu S + \frac{1}{2}\sigma^2 S)dt + \sigma S dZ \tag{11}$$

There are many criteria that can be used to measure the accuracy of predicted predictions. Mean Absolute Percentage Error (MAPE) is one of the commonly used error measures in measuring the consistency of the estimation results; calculations for this criterion are made in Microsoft Excel environment and the results are shown in the tables below. In the literature, MAPE statistic has a meaning as a percentage because of its estimation errors as a percentage compared to other methods, which makes it more accepted in practice. Accordingly, the method; εt error value, estimation value, σt actual value, N to indicate the number of observations, is defined as follows (Poon, 2005:23);

$$MAPE = \frac{1}{N}\sum_{t=1}^{N}\frac{|\varepsilon_t|}{\sigma_t} = \frac{1}{N}\sum_{t=1}^{N}\frac{|\hat{\sigma}_t - \sigma_t|}{\sigma_t} \quad (12)$$

Table 1. Accuracy Criteria Comparison of Prediction Error

MAPE	Prediction Accuracy Statement
< % 10	Very Good
% 10 - % 20	Good Predicton
% 20 - % 50	Acceptable
>% 50	False and incorrect

Source: Abidin and Jaffar (2014:109)

Abidin and Jaffar (2014) found that models with MAPE values below 10% were "very good", models with 10% to 20% were "good", models with 20% to 50% were "acceptable" and 50% the models on the "false and incorrect" classified.

2. Analysis of Data

GBM's formula is created in Excel worksheet and modeled for new prices;

$$S(t) = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right] \quad (13)$$

Below is a description of the parameters that make up the formula.

$S(t) =$ Value of stock at time t,

$S_0 =$ Initial value of stock,

$\exp =$ Exponential function,

$\mu =$ Return rate of stock,

$\sigma =$ Standard deviation of stock,

$\Delta t =$ time period,

$W(t) = \varepsilon.\sqrt{dt} =$ Wiener Process,

$\varepsilon =$ random number from the standard normal distribution table with a standard deviation of 1 and a mean of 0.

In the simulation of daily share price changes, it is calculated as $\Delta t =$ 0,0397 (1/252) and based on the study. 252 as the reason for a year consists of 252 business days.

The random value ε (epsilon) in the formula is generated separately for the indices. In the generation of these numbers, NORMTERS function, which is a random number generation function with normal distribution, is used in Excel. The simulation values for the application of the GBM simulation to the shares are given in Table 3 below.

The daily closing prices of 14 company shares in the metal industry, which is one of the sectors with the highest stock performance between 04.01.2010 and 30.11.2018, were used for the non-sample forecast for the period of one month by using the daily closing prices for the application. The return of daily data was calculated logarithmically. The calculation of logarithmic returns and their preference in this analysis is to avoid the negative effects of extreme values. The companies that have simulated shares are listed in Table 2.

Table 2. Companies are Used in the Study

Code	Company Name
BRSAN	Borusan Mannesmann Boru Sanayi ve Ticaret A.Ş.
BURCE	Buçelik Bursa Çelik Döküm Sanayi A.Ş.
BURVA	Burçelik Vana Sanayi ve Ticaret A.Ş.
CELHA	Çelik Halat ve Tel Sanayi A.Ş.
CEMTS	Çemtaş Çelik Makine Sanayi ve Ticaret A.Ş.
DMSAS	Demisaş Döküm Emaye Mamülleri Sanayi A.Ş.
DOKTA	Döktaş Dökümcülük Ticaret ve Sanayi A.Ş.
ERBOS	Erbosan Erciyas Boru Sanayi ve Ticaret A.Ş.
EREGL	Ereğli Demir ve Çelik Fabrikaları T.A.Ş.
IZMDC	İzmir Demir Çelik Sanayi A.Ş.
KRDM	Kardemir Karabük Demir Çelik Sanayi ve Ticaret A.Ş.
SARKY	Sarkuysan Eletrolitik Bakır Sanayi ve Ticaret A.Ş.

In order to compare the predictive performances of this model, MAPE statistics were calculated over a 30-day period. These statistical prediction criteria are used to measure the difference between the predicted return and the realized return. These criteria should be as small as possible for the selection of the most successful prediction model. The MAPE statistics calculated for the GBM model are given below.

Table 3: Estimated Results and MAPE Values for Main Metal Industry Companies

BRSAN				BURCE			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	7,22	7,23	0,0011	3.12.2018	3,15	3,16	0,0027
4.12.2018	7,06	7,19	0,0182	4.12.2018	3,17	3,14	0,0098
5.12.2018	7,19	7,15	0,0055	5.12.2018	3,16	3,16	0,0005
6.12.2018	7,12	7,15	0,0049	6.12.2018	3,2	3,17	0,0095
7.12.2018	7,19	7,19	0,0004	7.12.2018	3,2	3,17	0,0083
10.12.2018	7,13	7,22	0,0126	10.12.2018	3,14	3,17	0,0081
11.12.2018	7,14	7,22	0,0107	11.12.2018	3,11	3,16	0,0169
12.12.2018	6,92	7,25	0,0482	12.12.2018	3,1	3,15	0,0172
13.12.2018	6,96	7,25	0,0413	13.12.2018	3,12	3,12	0,0008
14.12.2018	6,78	7,3	0,0762	14.12.2018	3,05	3,15	0,0321
17.12.2018	6,74	7,3	0,0831	17.12.2018	3,05	3,13	0,0257
18.12.2018	6,75	7,26	0,0762	18.12.2018	3,05	3,1	0,0153
19.12.2018	6,81	7,26	0,0654	19.12.2018	3,12	3,1	0,008
20.12.2018	6,95	7,2	0,0362	20.12.2018	3,31	3,08	0,0694
21.12.2018	7	7,21	0,0301	21.12.2018	3,22	3,06	0,0486
24.12.2018	7,09	7,17	0,0112	24.12.2018	3,18	3,07	0,034
25.12.2018	7,01	7,14	0,0182	25.12.2018	3,1	3,08	0,0068
26.12.2018	7,05	7,11	0,0081	26.12.2018	3,09	3,07	0,0068
27.12.2018	6,99	7,11	0,0168	27.12.2018	3,07	3,08	0,0028
28.12.2018	6,97	7,13	0,0222	28.12.2018	3,09	3,1	0,003
31.12.2018	7,01	7,13	0,017	31.12.2018	3,21	3,08	0,0394
MAPE (%)		2,87		**MAPE (%)**		1,74	
BURVA				**CELHA**			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	1,7	1,68	0,0146	3.12.2018	4,93	4,94	0,0016
4.12.2018	1,73	1,67	0,0325	4.12.2018	4,84	4,96	0,0253
5.12.2018	1,71	1,68	0,019	5.12.2018	4,84	4,96	0,0255
6.12.2018	1,67	1,67	0,0022	6.12.2018	4,81	4,95	0,0301
7.12.2018	1,67	1,68	0,0038	7.12.2018	4,77	4,95	0,0378
10.12.2018	1,65	1,67	0,0142	10.12.2018	4,77	4,96	0,0404
11.12.2018	1,66	1,67	0,0074	11.12.2018	4,78	4,91	0,0281
12.12.2018	1,6	1,67	0,0434	12.12.2018	4,62	4,86	0,0528

13.12.2018	1,61	1,67	0,0385	13.12.2018	4,7	4,88	0,0378
14.12.2018	1,55	1,66	0,0682	14.12.2018	4,7	4,89	0,041
17.12.2018	1,57	1,64	0,044	17.12.2018	4,72	4,89	0,0363
18.12.2018	1,58	1,64	0,0383	18.12.2018	4,78	4,86	0,0171
19.12.2018	1,57	1,64	0,0419	19.12.2018	4,8	4,85	0,0109
20.12.2018	1,62	1,63	0,0084	20.12.2018	4,76	4,85	0,0188
21.12.2018	1,59	1,63	0,0256	21.12.2018	4,73	4,87	0,0286
24.12.2018	1,57	1,64	0,0445	24.12.2018	4,73	4,84	0,023
25.12.2018	1,54	1,62	0,0547	25.12.2018	4,62	4,84	0,0467
26.12.2018	1,52	1,62	0,0638	26.12.2018	4,77	4,81	0,0094
27.12.2018	1,5	1,62	0,0786	27.12.2018	4,81	4,76	0,0095
28.12.2018	1,46	1,63	0,1152	28.12.2018	5,28	4,76	0,0983
31.12.2018	1,48	1,63	0,0984	31.12.2018	5,4	4,76	0,1188
MAPE (%)		4,08		**MAPE (%)**		3,51	

CEMTS				**DMSAS**			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	7,11	6,93	0,0246	3.12.2018	2,69	2,69	0,0004
4.12.2018	6,92	6,92	0,0007	4.12.2018	2,64	2,67	0,0109
5.12.2018	6,9	6,86	0,0064	5.12.2018	2,64	2,65	0,0051
6.12.2018	6,71	6,84	0,0188	6.12.2018	2,61	2,65	0,0169
7.12.2018	6,66	6,75	0,0134	7.12.2018	2,61	2,65	0,0144
10.12.2018	6,56	6,75	0,0283	10.12.2018	2,59	2,63	0,0167
11.12.2018	6,52	6,76	0,0365	11.12.2018	2,52	2,63	0,044
12.12.2018	6,3	6,71	0,0652	12.12.2018	2,36	2,6	0,1026
13.12.2018	6,41	6,7	0,0456	13.12.2018	2,41	2,61	0,0843
14.12.2018	6,26	6,72	0,0735	14.12.2018	2,32	2,6	0,1202
17.12.2018	6,22	6,7	0,0771	17.12.2018	2,28	2,59	0,135
18.12.2018	6,31	6,65	0,0531	18.12.2018	2,33	2,58	0,1069
19.12.2018	6,44	6,69	0,0394	19.12.2018	2,33	2,58	0,1082
20.12.2018	6,35	6,66	0,0481	20.12.2018	2,38	2,6	0,0915
21.12.2018	6,24	6,68	0,071	21.12.2018	2,43	2,61	0,0746
24.12.2018	6,12	6,69	0,0937	24.12.2018	2,43	2,6	0,0702
25.12.2018	6,02	6,7	0,1133	25.12.2018	2,38	2,58	0,0857
26.12.2018	5,9	6,68	0,1323	26.12.2018	2,32	2,59	0,116
27.12.2018	5,83	6,65	0,14	27.12.2018	2,32	2,6	0,1193
28.12.2018	5,8	6,68	0,1521	28.12.2018	2,29	2,57	0,1221
31.12.2018	6,08	6,68	0,0981	31.12.2018	2,3	2,57	0,1165
MAPE (%)		6,34		**MAPE (%)**		7,44	

DOKTA				**ERBOS**			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	8,8	8,36	0,05	3.12.2018	16,14	16,1	0,0025
4.12.2018	8,52	8,32	0,0231	4.12.2018	15,84	15,98	0,0088
5.12.2018	8,59	8,38	0,0248	5.12.2018	16,02	15,86	0,0102
6.12.2018	8,49	8,35	0,0165	6.12.2018	15,91	15,88	0,0018
7.12.2018	8,48	8,4	0,0091	7.12.2018	15,91	16	0,0053
10.12.2018	8,38	8,34	0,0046	10.12.2018	15,78	15,83	0,0032
11.12.2018	8,06	8,36	0,0368	11.12.2018	15,66	15,9	0,0154

12.12.2018	7,84	8,37	0,0676	12.12.2018	15,27	15,76	0,0321
13.12.2018	8,17	8,39	0,0267	13.12.2018	15,41	15,91	0,0328
14.12.2018	7,8	8,43	0,0814	14.12.2018	15,02	15,96	0,0628
17.12.2018	7,97	8,44	0,0594	17.12.2018	15,11	15,83	0,0474
18.12.2018	8,29	8,5	0,0248	18.12.2018	15,12	15,89	0,0506
19.12.2018	8,5	8,49	0,0011	19.12.2018	15,35	15,84	0,0318
20.12.2018	8,57	8,53	0,0051	20.12.2018	15,45	15,88	0,0275
21.12.2018	8,69	8,5	0,0214	21.12.2018	15,55	15,97	0,0269
24.12.2018	8,68	8,47	0,0248	24.12.2018	15,48	15,9	0,0272
25.12.2018	8,48	8,51	0,0038	25.12.2018	15,24	15,8	0,0365
26.12.2018	8,62	8,55	0,0077	26.12.2018	15,31	15,78	0,0307
27.12.2018	8,29	8,54	0,0305	27.12.2018	15,23	15,77	0,0351
28.12.2018	8,34	8,5	0,0188	28.12.2018	15,12	15,7	0,0384
31.12.2018	8,15	8,52	0,0456	31.12.2018	15,2	15,66	0,0304
MAPE (%)		2,78		**MAPE (%)**		2,65	

EREGL				**IZMDC**			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	6,87	6,38	0,0709	3.12.2018	1,8	1,79	0,0052
4.12.2018	6,67	6,41	0,0389	4.12.2018	1,78	1,78	0,0025
5.12.2018	6,96	6,39	0,0825	5.12.2018	1,78	1,79	0,0034
6.12.2018	6,59	6,38	0,0314	6.12.2018	1,76	1,77	0,0058
7.12.2018	6,62	6,39	0,034	7.12.2018	1,79	1,77	0,0136
10.12.2018	6,28	6,34	0,0091	10.12.2018	1,77	1,76	0,0047
11.12.2018	6,26	6,34	0,0126	11.12.2018	1,78	1,77	0,0068
12.12.2018	6,22	6,37	0,0245	12.12.2018	1,69	1,76	0,0424
13.12.2018	6,56	6,36	0,0308	13.12.2018	1,67	1,76	0,0523
14.12.2018	6,37	6,31	0,0102	14.12.2018	1,66	1,74	0,0484
17.12.2018	6,35	6,29	0,0094	17.12.2018	1,64	1,73	0,056
18.12.2018	6,32	6,29	0,004	18.12.2018	1,67	1,74	0,04
19.12.2018	6,46	6,28	0,0284	19.12.2018	1,69	1,73	0,023
20.12.2018	6,29	6,28	0,0009	20.12.2018	1,68	1,73	0,0325
21.12.2018	6,27	6,26	0,0014	21.12.2018	1,69	1,72	0,0205
24.12.2018	6,27	6,22	0,0078	24.12.2018	1,69	1,72	0,0199
25.12.2018	6,19	6,2	0,0014	25.12.2018	1,68	1,74	0,0367
26.12.2018	6,14	6,23	0,0144	26.12.2018	1,67	1,74	0,0406
27.12.2018	6,01	6,24	0,0377	27.12.2018	1,67	1,75	0,047
28.12.2018	6,01	6,21	0,0326	28.12.2018	1,65	1,75	0,0622
31.12.2018	6,1	6,2	0,0159	31.12.2018	1,66	1,74	0,0502
MAPE (%)		2,37		**MAPE (%)**		2,92	

KRDMA				**KRDMB**			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	2,06	2,04	0,0121	3.12.2018	2,05	1,99	0,0268
4.12.2018	1,99	2,04	0,0227	4.12.2018	1,98	1,99	0,0032
5.12.2018	1,99	2,02	0,0171	5.12.2018	2	1,99	0,0063
6.12.2018	1,98	2,02	0,0189	6.12.2018	1,98	2	0,0083
7.12.2018	2	2	0,0012	7.12.2018	1,99	1,99	0,0009
10.12.2018	1,93	1,99	0,0324	10.12.2018	1,94	1,98	0,0216

11.12.2018	1,91	1,98	0,0388	11.12.2018	1,92	1,99	0,034
12.12.2018	1,83	2	0,0911	12.12.2018	1,83	1,97	0,0765
13.12.2018	1,84	2	0,0845	13.12.2018	1,84	1,97	0,0728
14.12.2018	1,78	1,99	0,1155	14.12.2018	1,78	1,97	0,1042
17.12.2018	1,76	1,99	0,1292	17.12.2018	1,75	1,95	0,1148
18.12.2018	1,77	1,96	0,107	18.12.2018	1,77	1,94	0,0962
19.12.2018	1,84	1,96	0,0654	19.12.2018	1,85	1,93	0,0459
20.12.2018	1,83	1,96	0,0733	20.12.2018	1,84	1,94	0,0543
21.12.2018	1,85	1,97	0,0664	21.12.2018	1,85	1,93	0,0426
24.12.2018	1,85	1,96	0,0604	24.12.2018	1,84	1,93	0,0491
25.12.2018	1,83	1,97	0,0755	25.12.2018	1,83	1,93	0,0551
26.12.2018	1,82	1,98	0,0876	26.12.2018	1,82	1,92	0,0565
27.12.2018	1,81	1,97	0,0885	27.12.2018	1,81	1,93	0,066
28.12.2018	1,79	1,97	0,1003	28.12.2018	1,79	1,93	0,0796
31.12.2018	1,82	1,96	0,0776	31.12.2018	1,83	1,94	0,0581
MAPE (%)		6,5		**MAPE (%)**		5,11	
KRDMD				**SARKY**			
Date	**Real Value**	**GBM**	**Absolute Error / Real Value**	**Date**	**Real Value**	**GBM**	**Absolute Error / Real Value**
3.12.2018	2,62	2,53	0,0337	3.12.2018	3,11	3,22	0,0346
4.12.2018	2,44	2,52	0,0341	4.12.2018	3,04	3,2	0,0512
5.12.2018	2,45	2,52	0,0269	5.12.2018	3,04	3,18	0,0476
6.12.2018	2,43	2,52	0,0374	6.12.2018	3	3,17	0,058
7.12.2018	2,42	2,5	0,0338	7.12.2018	2,99	3,18	0,0625
10.12.2018	2,28	2,49	0,0905	10.12.2018	2,98	3,17	0,0639
11.12.2018	2,3	2,5	0,0858	11.12.2018	3,12	3,19	0,0222
12.12.2018	2,19	2,49	0,1361	12.12.2018	3,13	3,18	0,0148
13.12.2018	2,26	2,49	0,1021	13.12.2018	3,14	3,16	0,0056
14.12.2018	2,15	2,49	0,1568	14.12.2018	3,19	3,16	0,0094
17.12.2018	2,13	2,47	0,1579	17.12.2018	3,14	3,14	0,0001
18.12.2018	2,19	2,45	0,1207	18.12.2018	3,15	3,16	0,0031
19.12.2018	2,3	2,46	0,0698	19.12.2018	3,19	3,15	0,0112
20.12.2018	2,29	2,46	0,0736	20.12.2018	3,2	3,17	0,0097
21.12.2018	2,3	2,46	0,0678	21.12.2018	3,17	3,19	0,005
24.12.2018	2,23	2,43	0,0896	24.12.2018	3,2	3,18	0,0067
25.12.2018	2,21	2,42	0,0954	25.12.2018	3,17	3,2	0,0083
26.12.2018	2,19	2,4	0,0942	26.12.2018	3,17	3,21	0,0116
27.12.2018	2,18	2,4	0,0994	27.12.2018	3,14	3,19	0,015
28.12.2018	2,14	2,4	0,1222	28.12.2018	3,12	3,18	0,0207
31.12.2018	2,18	2,4	0,102	31.12.2018	3,14	3,19	0,0155
MAPE (%)		8,71		**MAPE (%)**		2,27	

When the estimates in the above table are examined, it is observed that the estimates presented with the actual data are close to the actual values. It's able to have knowledge about accuracy of predictions according to MAPE criteria. For 14 companies, the MAPE error value of the GBM model is less than 10% and is reasonable for estimation according to Table 1.

According to all these results; It can be said that the predictions made by the GBM model are consistent and close to realistic. According to the values obtained as a result of the application; It is seen that GBM model has high prediction performance and makes less prediction errors.

3. Conclusion

The securities markets and stock exchanges are seen as an important part of the economy. Because stock markets play an important role in the development of a country's industry and trade and thus affect their economy to a great extent. At the same time, the size and efficiency of stock exchanges provide information to researchers and investors about the state of the economy of the countries. Therefore, it can be said that the main reason why the government, industry and even the central banks of the country closely follow the events in the stock market is this important function of the stock exchanges. They have an important place in stock markets, governments and companies as well as investors.

For the investors, while the stock market is a point of gain, sometimes there is a loss of gains. In order to avoid losing, reducing or increasing their gains, investors try to estimate the stock prices by various methods.

In this respect, the aim of our study; is to seek answers to the question of what will happen in a given period of stock pricing. In order to answer this question, GBM, which is the most widely used model to randomly model financial markets, has been used. The best estimate we can make of the future is that the stochastic process, which is only the latest data, is used to model the price movement of a risky asset.

In the financial market, investors compare financial assets with returns rather than prices. The most commonly used basic assumption about the behavior of these returns is that they have normal distribution. The logarithmic return of the stock at any time t is considered to have a normal distribution. The fact that the stock price cannot be negative requires the use of GBM to model the price movement. Furthermore, a lognormally distributed continuous time stochastic process with independent changes can only be explained by GBM.

It is possible to model the price of financial assets for the next few years or even a few minutes with GBM. As a result; GBM is one of the most widely used methods for estimating the movement of financial markets in an uncertainty environment. The most important part in the GBM Modeling process is the Wiener Process which is the source of randomness. Here, the random generation of random numbers from the normal distribution table prevents external intervention in the model and the market is thus imitated.

In the application, the daily closing prices of 14 company shares in the main metal industry, which is one of the sectors with the highest stock performance between the dates of 04.01.2010 and 30.11.2018, were used for the non-sample forecast for the period of one month by using the daily closing prices.

The stock price estimations made by the GBM method were compared by comparing the non-sample forecasts made for the one-month period between 03.12.2018-31.12.2018 and the performance was evaluated. In order to compare the predictive performances of this model, MAPE statistics were calculated over a 30-day period. This statistical prediction criterion is used to measure the difference between the predicted return and actual return. This criterion should be as small as possible for the selection of the most successful prediction model.

As a result of the application; It is observed that the estimates presented together with the actual data are close to the actual values. It's able to have knowledge about the accuracy of predictions according to MAPE criteria. For 14 companies, the MAPM error value of the GBM model is less than 10% and is reasonable for estimation according to Table 1. According to all these results; It can be said that the predictions made by the CPD model are consistent and close to realistic.

As a result; GBM is one of the most widely used methods for estimating the movement of financial markets in uncertainty environment, which shows us how prices can move and which limits can be moved with a scientific approach in financial and commercial transactions in an uncertainty environment. It will also be useful to establish and interpret the model for short-term terms.

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